Three trigonometric ratios in a right-angled triangle are defined as:

\[ \sin \alpha = \frac{\text{Opposite side}}{\text{Hypotenuse}} \]

\[ \cos \alpha = \frac{\text{Adjacent side}}{\text{Hypotenuse}} \]

\[ \tan \alpha = \frac{\text{Opposite side}}{\text{Adjacent side}} \]

You can use the acronym **SOHCAHTOA** to help you remember which ratio is which.

- **SOH** as \( \sin \alpha = \frac{O}{H} \)
- **CAH** as \( \cos \alpha = \frac{A}{H} \)
- **TOA** as \( \tan \alpha = \frac{O}{A} \)
For each triangle, write down the three trigonometric ratios for the angle $\theta$ in terms of the sides of the triangle.

**Answers**

\[ a \quad \sin \theta = \frac{AB}{AC}, \quad \cos \theta = \frac{BC}{AC}, \quad \tan \theta = \frac{AB}{BC} \]

\[ b \quad \sin \theta = \frac{BC}{AC}, \quad \cos \theta = \frac{AB}{AC}, \quad \tan \theta = \frac{BC}{AB} \]
Find the length of the unknown sides in triangle ABC. Give your answer to 3sf.

**Answer**

To find BC:

\[ \cos 30^\circ = \frac{BC}{8} \]

\[ BC = 8 \cos 30^\circ \]

\[ BC = 6.93 \text{ cm (to 3 sf)} \]
Finding the angles of a right-angled triangle

If you know the lengths of two sides in a right-angled triangle, you can find

- the length of the other side by using Pythagoras
- the size of the two acute angles by using the appropriate trigonometric ratios.
Find the angle marked $\theta$ in each triangle.
Give your answers correct to the nearest degree.

**Answers**

**a**  
\[ \tan \theta = \frac{8}{5} \]
\[ \theta = \tan^{-1} \left( \frac{8}{5} \right) \]
\[ \theta = 58^\circ \]

**b**  
\[ \sin \theta = \frac{3}{6.5} \]
\[ \theta = \sin^{-1} \left( \frac{3}{6.5} \right) \]
\[ \theta = 27^\circ \]
Triangle ABC is isosceles. The two equal sides AB and BC are 10 cm long and each makes an angle of 40° with AC.

a. Represent this information in a clear and labeled diagram.

b. Find the length of AC.

c. Find the perimeter of triangle ABC.

**Answers**

\[ \cos 40° = \frac{AP}{10} \]

\[ AP = 10 \cos 40° \]

\[ AC = 2 \times 10 \cos 40° \]

\[ AC = 15.3 \text{ cm} \]

\[ \text{Perimeter} = AB + BC + CA \\
= 15.32 \ldots + 2 \times 10 \\
= 35.3 \text{ cm (to 3 sf)} \]
The diagonals of a rhombus are 10 cm and 5 cm. Find the size of the larger angle of the rhombus.

\[
\tan \text{angle OAB} = \frac{5}{2.5}
\]

Angle OAB = \tan^{-1}\left(\frac{5}{2.5}\right)

Angle BAD = 2 \times \text{OAB}

= 2 \times \tan^{-1}\left(\frac{5}{2.5}\right)

Angle BAD = 127^\circ \text{ (to 3 sf)}
Angles of elevation and depression

→ The **angle of elevation** is the angle you lift your eyes through to look at something above.

→ The **angle of depression** is the angle you lower your eyes through to look at something below.
From a yacht, 150 metres out at sea, the angle of elevation of the top of a cliff is $17^\circ$. The angle of elevation to the top of a lighthouse on the cliff is $20^\circ$. This information is shown in the diagram.

a Find the height of the cliff.

b Hence find the height of the lighthouse.

**Answers**

a Let $x$ be the height of the cliff

$$\tan 17^\circ = \frac{x}{150}$$

$x = 45.9 \text{ m (to 3 sf)}$

b Let $y$ be the distance from the top of the lighthouse to the sea.

$$\tan 20^\circ = \frac{y}{150}$$

$y = 54.5955\ldots \text{ m}$

height of the lighthouse $= y - x$

$= 8.74 \text{ m (to 3 sf)}$
A boy standing on a hill at X can see a boat on a lake at Y as shown in the diagram. The vertical distance from X to Y is 60 m and the horizontal distance is 100 m.

Find:

a  the shortest distance between the boy and the boat
b  the angle of depression of the boat from the boy.

**Answers**

a  \[ XY^2 = 100^2 + 60^2 \]
   \[ XY = 117 \text{ m (to 3 sf)} \]

b  \[ \tan \beta = \frac{60}{100} \]
   The angle of depression
   \[ = 31.0^\circ \text{ (to 3 sf)} \]