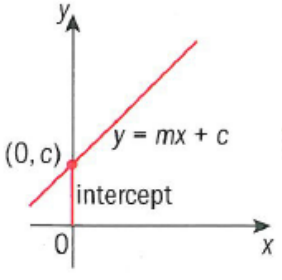
3.2 Equations of lines

→ The equation of a straight line can be written in the form $y = mx + c$, where

- m is the **gradient**
- c is the y-**intercept** (y-coordinate of the point where the line crosses the y-axis).

Values for variables x and y are said to **satisfy** an equation if, when the variables are replaced by the respective values, the two sides of the equation are equal.



Ex. 1

The line L passes through the point A(1, 7) and has gradient 5.
Find the equation of L.
Give your answer in the form $y = mx + c$.

x y

m

$y = 5x + c$

$7 = 5(1) + c$

$2 = c$

$y = 5x + 2$

The line L has gradient $\frac{1}{3}$ and passes through A$(2, -1)$.

a Find the equation of L. Give your answer in the form $y = mx + c$.

b Write down the point of intersection of L with the y-axis.

c Find the point of intersection of L with the x-axis.

d Draw the line L showing clearly the information found in **b** and **c**.

a. $y = mx+b \rightarrow -1 = \frac{1}{3}(2)+c$

$-1 = \frac{2}{3}+c \rightarrow$ $-\frac{2}{3}$ $-\frac{2}{3}$

$= \frac{-5}{3} = c$

$\boxed{y = \frac{1}{3}x - \frac{5}{3}}$

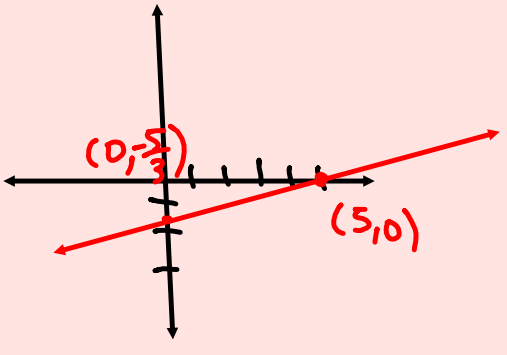
b. $\boxed{(0, -\frac{5}{3})}$

c. $0 = \frac{1}{3}x - \frac{5}{3}$ $\boxed{(5,0)}$

$\frac{5}{3} = \frac{1}{3}x$

$\times 3$ $\times 3$

$5 = x$



→ The equation of a straight line can be written in the form

$ax + by + c = 0$

where a, b and $c \in \mathbb{Z}$.

a, b & c are elements of the Integers

$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Ex3

a Line L joins the points A(–3, 5) and B(1, 2).
Find the equation of line L.
Give your answer in the form $ax + by + c = 0$ where $a, b, c \in \mathbb{Z}$

b The point $Q\left(\frac{5}{3}, t\right)$ lies on L. Find the value of t.

a. $m = \frac{5-2}{-3-1} = \frac{3}{-4}$

$y = mx + c$

$2 = \frac{-3}{4}(1) + c \rightarrow c = 2\frac{3}{4} = \frac{11}{4}$

$c = \frac{11}{4}$

$y = \frac{-3}{4}x + \frac{11}{4}$

$+\frac{3}{4}x \quad -\frac{11}{4} \quad +\frac{3}{4}x \quad -\frac{11}{4}$

$4\left(\frac{3}{4}x + y - \frac{11}{4} = 0\right)$

$3x + 4y - 11 = 0$

b. $y = \frac{-3}{4}x + \frac{11}{4}$

$t = \frac{-3}{4}\left(\frac{5}{3}\right) + \frac{11}{4}$

$= \frac{-5}{4} + \frac{11}{4}$

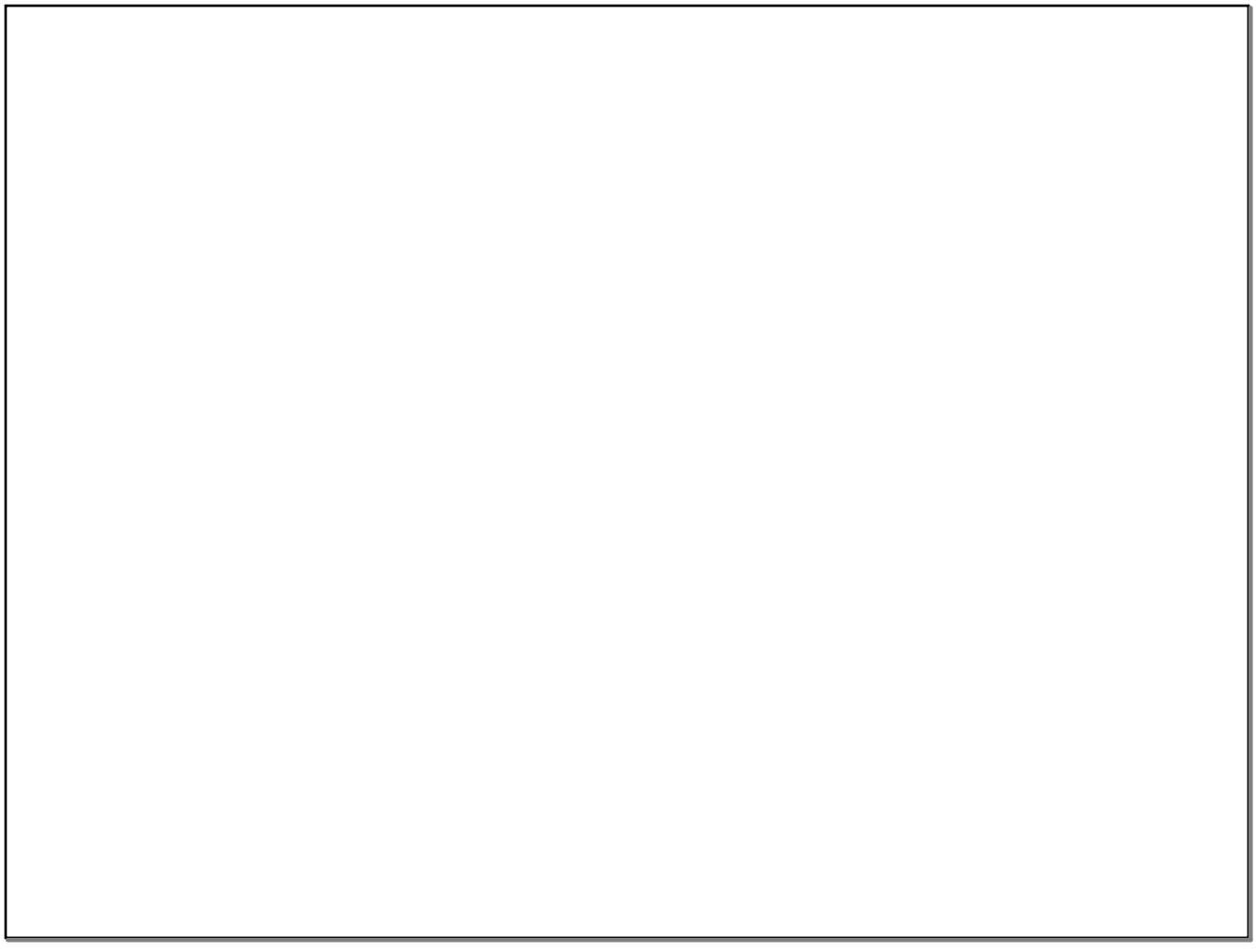
$= \frac{6}{4}$

$t = 3/2$

Vertical and horizontal lines

Vertical lines are parallel to the y-axis.
Horizontal lines are parallel to the x-axis.

- The equation of any vertical line is of the form $x = k$ where k is a constant.
- The equation of any horizontal line is of the form $y = k$ where k is a constant.

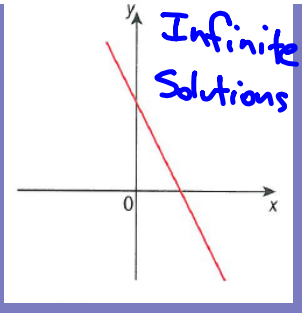


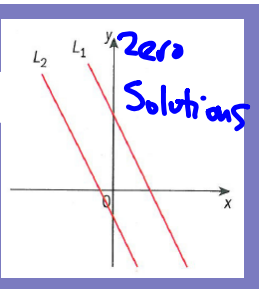
Intersection of lines in two dimensions

→ If two lines are parallel then they have the same gradient and do not intersect.

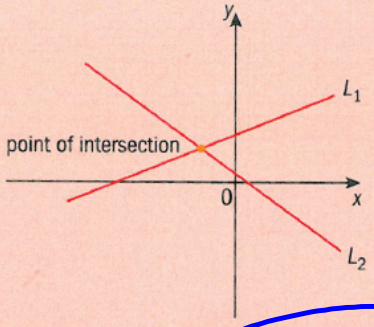
Parallel lines L_1 and L_2 can be:

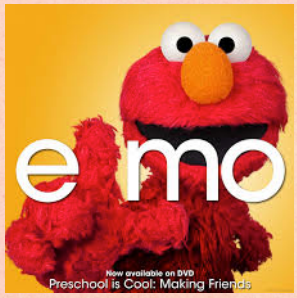
- *Coincident lines (the same line)*
- *Different lines*





→ If two lines L_1 and L_2 are not parallel then they intersect at just one point.





To find the point of intersection write $m_1x_1 + c_1 = m_2x_2 + c_2$ and solve for x.

Find the point of intersection of the lines $y = 2x + 1$ and $-x - y + 4 = 0$.

Y_1: $Y=2X+1$

Y_2: $Y=-X+4$

2nd Calc Int.

$-X-Y+4=0$

$-X+4=Y$

$(1,3)$