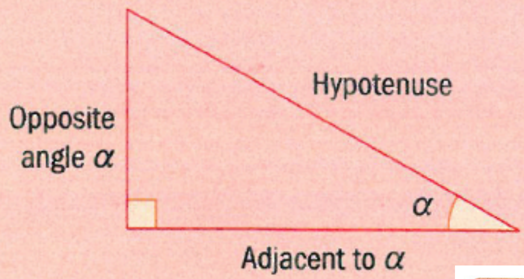
3.3 The sine, cosine and tangent ratios

→ Three trigonometric ratios in a right-angled triangle are defined as

$$\sin \alpha = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\cos \alpha = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

$$\tan \alpha = \frac{\text{Opposite side}}{\text{Adjacent side}}$$



You can use the acronym **SOHCAHTOA** to help you remember which ratio is which.

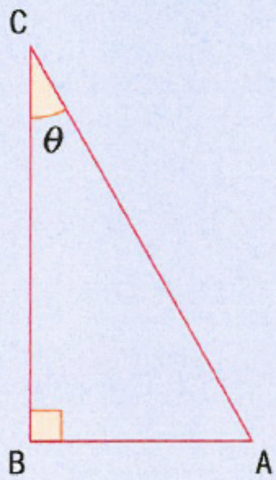
SOH as **S**in $\alpha = \frac{\mathbf{O}}{\mathbf{H}}$

CAH as **C**os $\alpha = \frac{\mathbf{A}}{\mathbf{H}}$

TOA as **T**an $\alpha = \frac{\mathbf{O}}{\mathbf{A}}$

For each triangle, write down the three trigonometric ratios for the angle θ in terms of the sides of the triangle.

a

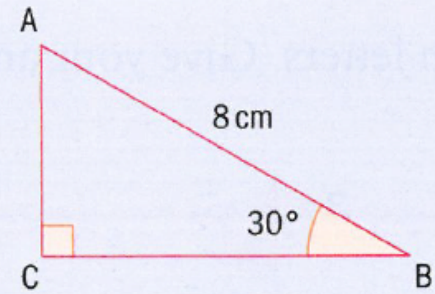


Answers

a $\sin\theta = \dfrac{AB}{AC}$, $\cos\theta = \dfrac{BC}{AC}$, $\tan\theta = \dfrac{AB}{BC}$

b $\sin\theta = \dfrac{BC}{AC}$, $\cos\theta = \dfrac{AB}{AC}$, $\tan\theta = \dfrac{BC}{AB}$

Find the length of the unknown sides in triangle ABC.
Give your answer to 3sf.



Answer

To find BC:

$$\cos 30° = \frac{BC}{8}$$

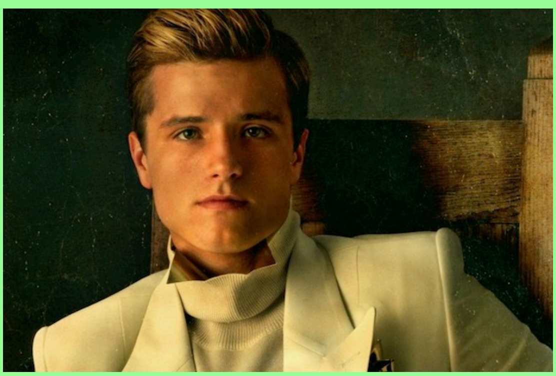
$$BC = 8\cos 30°$$

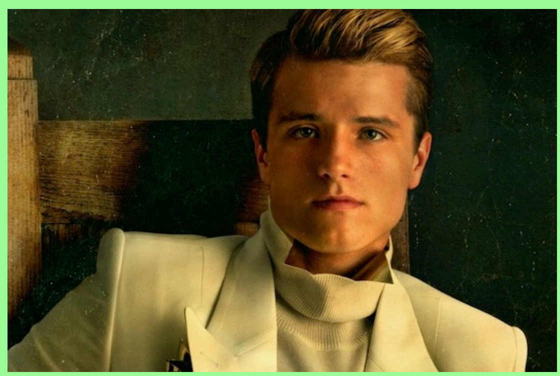
$$BC = 6.93 \text{ cm (to 3 sf)}$$

Finding the angles of a right-angled triangle

If you know the lengths of two sides in a right-angled triangle, you can find

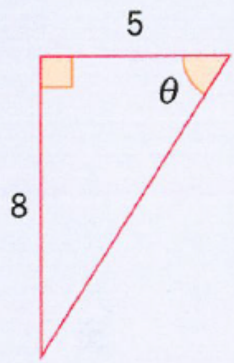
- the length of the other side by using Pythagoras
- the size of the two acute angles by using the appropriate trigonometric ratios.



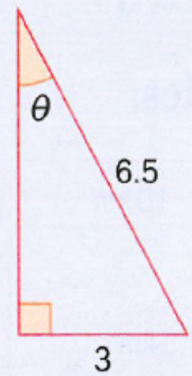


Find the angle marked θ in each triangle.
Give your answers correct to the nearest degree.

a



b



Answers

a $\tan\theta = \frac{8}{5}$

$\theta = \tan^{-1}\left(\frac{8}{5}\right)$

$\theta = 58°$

b $\sin\theta = \frac{3}{6.5}$

$\theta = \sin^{-1}\left(\frac{3}{6.5}\right)$

$\theta = 27°$

Triangle ABC is isosceles. The two equal sides AB and BC are 10 cm long and each makes an angle of 40° with AC.

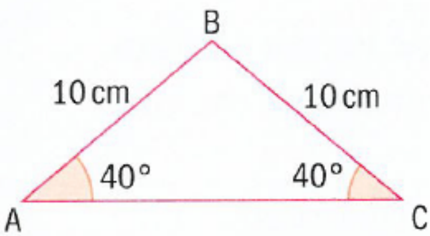
a Represent this information in a clear and labeled diagram.

b Find the length of AC.

c Find the perimeter of triangle ABC.

Answers

a



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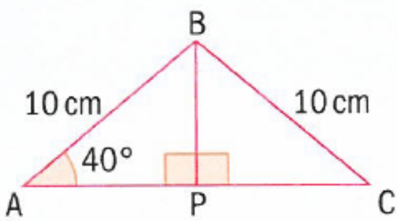


$$\cos 40^\circ = \frac{AP}{10}$$

$$AP = 10\cos 40^\circ$$

$$AC = 2 \times 10\cos 40^\circ$$

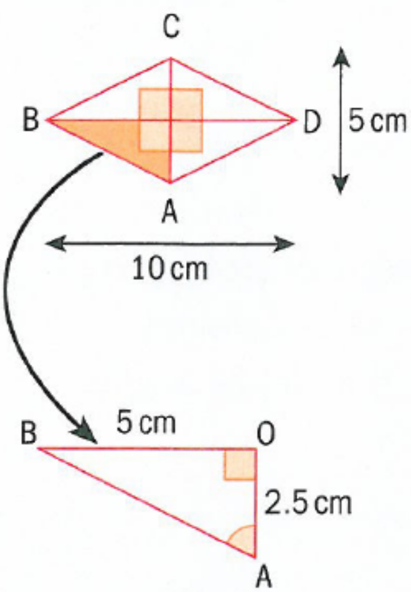
$$AC = 15.3\,\text{cm}$$

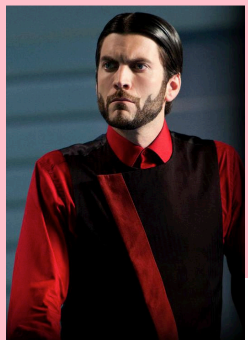
c

$$\begin{aligned}\text{Perimeter} &= AB + BC + CA \\ &= 15.32\ldots + 2 \times 10 \\ &= 35.3\,\text{cm (to 3 sf)}\end{aligned}$$

The diagonals of a rhombus are 10 cm and 5 cm. Find the size of the **larger** angle of the rhombus.

Answer





$$\tan \text{angle OAB} = \frac{5}{2.5}$$

$$\text{Angle OAB} = \tan^{-1}\left(\frac{5}{2.5}\right)$$

$$\text{Angle BAD} = 2 \times \text{OAB}$$

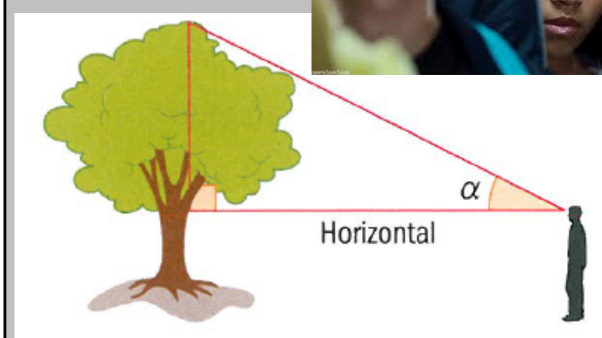
$$= 2 \times \tan^{-1}\left(\frac{5}{2.5}\right)$$

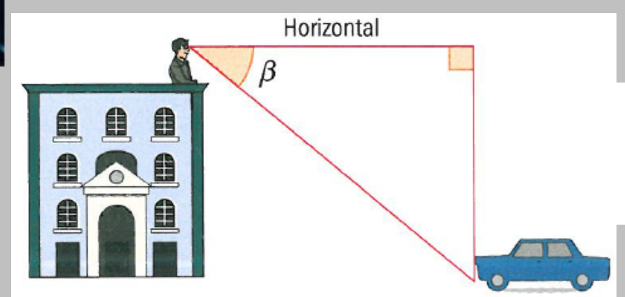
$$\text{Angle BAD} = 127^\circ \text{ (to 3 sf)}$$

Angles of elevation and depression

→ The **angle of elevation** is the angle you lift your eyes through to look at something above.

→ The **angle of depression** is the angle you lower your eyes through to look at something below.

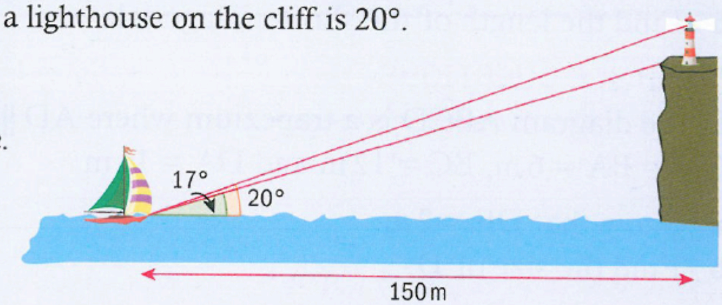




From a yacht, 150 metres out at sea, the angle of elevation of the top of a cliff is 17°. The angle of elevation to the top of a lighthouse on the cliff is 20°. This information is shown in the diagram.

a Find the height of the cliff.

b Hence find the height of the lighthouse.



Answers

a Let x be the height of the cliff

$$\tan 17° = \frac{x}{150}$$

$x = 45.9$ m (to 3 sf)

b Let y be the distance from the top of the lighthouse to the sea.

$$\tan 20° = \frac{y}{150}$$

$$y = 54.5955\ldots \text{m}$$

height of the lighthouse $= y - x$

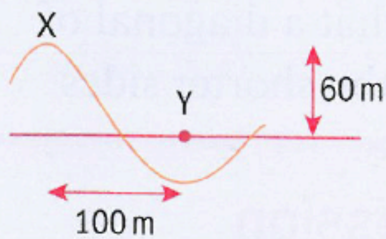
$= 8.74$ m (to 3 sf)

A boy standing on a hill at X can see a boat on a lake at Y as shown in the diagram. The vertical distance from X to Y is 60 m and the horizontal distance is 100 m.

Find:

a the shortest distance between the boy and the boat

b the angle of depression of the boat from the boy.



Answers

a $XY^2 = 100^2 + 60^2$

$XY = 117\,\text{m}$ (to 3 sf)

b $\tan\beta = \dfrac{60}{100}$

The angle of depression $= 31.0°$ (to 3 sf)