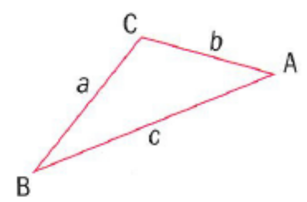
3.4 The sine and cosine rules

The sine and cosine rules are formulae that will help you to find unknown sides and angles in a triangle. They enable you to use trigonomentry in triangles that are **not** right-angled.

The formula and notation are simpler if you label triangles like this.

- The side opposite $\hat{A}$ is a.
- The side opposite $\hat{B}$ is b.
- The side opposite $\hat{C}$ is c.

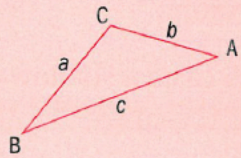


Sine rule

In any triangle ABC with angles $\hat{A}$, $\hat{B}$ and $\hat{C}$, and opposite sides a, b and c respectively:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



The sine rule is in the Formula booklet.

In triangle ABC, $b = 16\,\text{cm}$, $c = 10\,\text{cm}$ and $\hat{B} = 135°$.

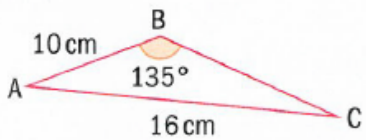
a Represent the given information in a labeled diagram.

b Find the size of angle C.

c Hence find the size of angle A.

Answers

a



b $\dfrac{16}{\sin 135°} = \dfrac{10}{\sin \hat{C}}$

$16 \sin \hat{C} = 10 \sin 135°$

$\sin \hat{C} = \dfrac{10\sin 135°}{16}$

$\hat{C} = 26.2°$ (to 3 sf)

c $\hat{A} + \hat{B} + \hat{C} = 180°$

$\hat{A} + 135° + 26.227\ldots = 180°$

$\hat{A} = 18.8°$ (to 3 sf)

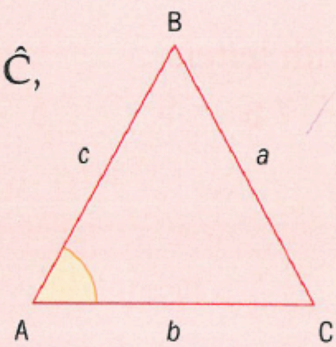
→ Cosine rule

In any triangle ABC with angles $\hat{A}$, $\hat{B}$ and $\hat{C}$, and opposite sides a, b and c respectively:

$$a^2 = b^2 + c^2 - 2bc\cos\hat{A}$$

This formula can be rearranged to

$$\cos\hat{A} = \frac{b^2 + c^2 - a^2}{2bc}$$



These formulae are in the Formula booklet. The first version of the formula is useful when you need to find a side. The second version of the formula is useful when you need to find an angle.

In triangle ABC, AC = 8.6 m, AB = 6.3 m and $\hat{A} = 50°$.
Find the length of BC.

Answer

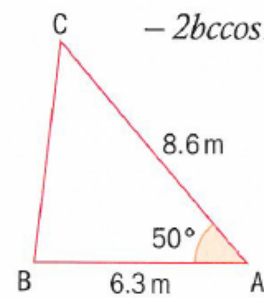
$BC^2 = 8.6^2 + 6.3^2 - 2 \times 8.6 \times 6.3 \times \cos 50°$

$BC^2 = 43.9975\ldots$

BC = 6.63 m (to 3 sf)

Sketch the triangle.

Use $a^2 = b^2 + c^2 - 2bc\cos\hat{A}$



X, Y and Z are three towns. X is 20 km due north of Z.
Y is to the east of line XZ. The distance from Y to X is 16 km and the distance from Z to Y is 8 km.

a Represent this information in a clear and labeled diagram.

b Find the size of angle X.

Answers

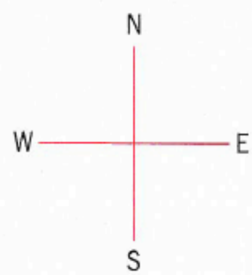
a

b $\cos \hat{X} = \dfrac{20^2 + 16^2 - 8^2}{2 \times 20 \times 16}$

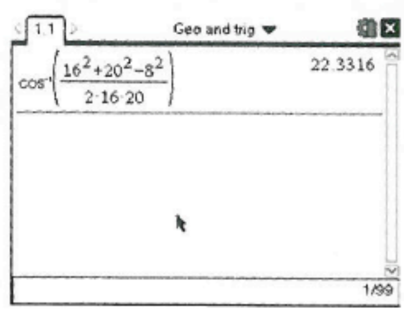
$$\hat{X} = \cos^{-1}\left(\frac{20^2 + 16^2 - 8^2}{2 \times 20 \times 16}\right)$$

$$= 22.3 \text{ (to 3 sf)}$$

Remember:



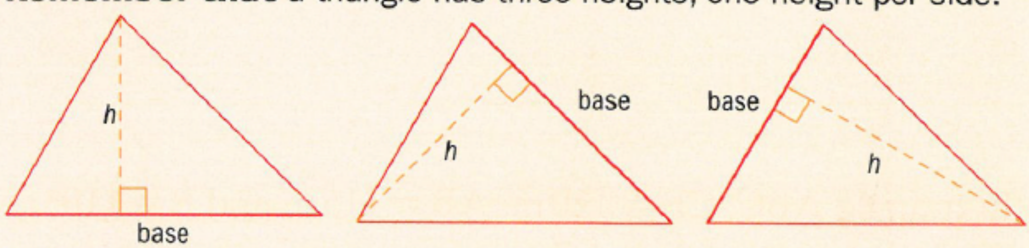
Use $\cos \hat{X} = \dfrac{y^2 + z^2 - x^2}{2yz}$



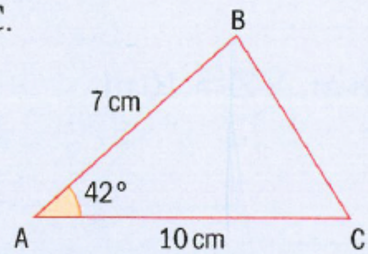
Area of a triangle

$$A = \frac{1}{2}(b \times h)$$

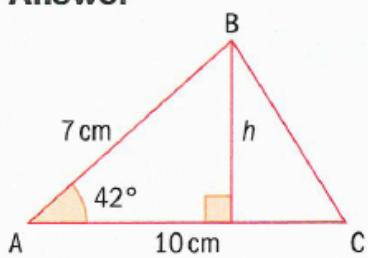
Remember that a triangle has three heights, one height per side.



Calculate the area of triangle ABC.



Answer



Use the formula $A = \frac{1}{2}(b \times h)$ *with AC as the base,* $b = 10$

Draw the height, h, the perpendicular to AC from B.

$$\sin 42° = \frac{h}{7} \Rightarrow h = 7\sin 42°$$

Substitute in the formula for the area of a triangle.

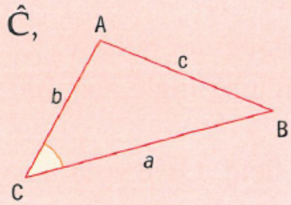
$$Area = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2}(10 \times 7\sin 42°)$$

$$= 23.4\,\text{cm}^2 \text{ (to 3 sf)}$$

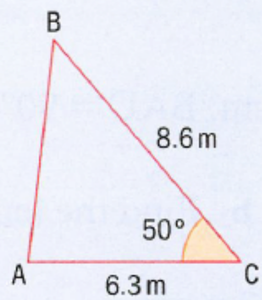
→ In any triangle ABC with angles $\hat{A}$, $\hat{B}$ and $\hat{C}$, and opposite sides a, b and c respectively, this rule applies:

Area of triangle $= \frac{1}{2}ab\sin\hat{C}$



This formula is in the formula booklet.

Calculate the area of the triangle ABC.



Answer

Area of triangle ABC =

$\frac{1}{2} \times 8.6 \times 6.3 \times \sin 50°$

$= 20.8\ m^2$ (to 3sf)

Substitute in the formula

$Area = \frac{1}{2}ab \sin\hat{C}$